

Seat No.	
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Total No. of Questions : 8]

[Total No. of Printed Pages : 4

[4261]-1

F. E. Examination - 2012

ENGINEERING MATHEMATICS - I

(2012 Pattern)

Time : 2 Hours]

[Max. Marks : 50

Instructions :

- (1) Attempt **four** questions : Q. No. 1 or 2, Q. No. 3 or 4, Q. No. 5 or 6 and Q. No. 7 or 8.
- (2) Figures to the right indicate full marks.
- (3) Assume suitable data if necessary.
- (4) Neat diagrams must be drawn wherever necessary.
- (5) Use of electronic non-programmable calculator is allowed.

Q.1) (A) Examine the consistency of the system of the following equations. If consistent, solve system of equations : [04]

$$x + y + z = 3$$

$$x + 2y + 3z = 4$$

$$x + 4y + 9z = 6$$

(B) Find Eigen Values and Eigen Vector corresponding to highest Eigen Value for the matrix : [04]

$$A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

- (C) If $\tan \log (x - iy) = a - ib$ and $a^2 + b^2 \neq 1$, then prove that $\tan \log (x^2 + y^2) = \frac{2a}{1 - a^2 - b^2}$. [04]

OR

- Q.2) (A) Examine for Linear Dependence or Independence of Vectors $(2, 2, 7, -1)$, $(3, -1, 2, 4)$ and $(1, 1, 3, 1)$. [04]

- (B) Solve : $x^7 + x^4 + i(x^3 + 1) = 0$. [04]

- (C) A square lies above real axis in Argand diagram and two of its adjacent vertices are the origin and the point $2 + 3i$. Find the complex number representing other vertices. [04]

- Q.3) (A) Test convergence of the series : (Any One) [04]

(a) $\frac{2}{9} + \frac{2 \cdot 5}{9 \cdot 13} + \frac{2 \cdot 5 \cdot 8}{9 \cdot 13 \cdot 17} + \dots$

(b) $\sum_{n=1}^{\infty} \left(\frac{1}{n} \right) \sin \left(\frac{1}{n} \right)$

- (B) Expand $3x^3 - 2x^2 + x - 6$ in powers of $(x - 2)$. [04]

- (C) Find n^{th} derivative of $x^2 e^x \cos x$. [04]

OR

- Q.4) (A) Solve : (Any One) [04]

(a) $\lim_{x \rightarrow 1} (1 - x^2)^{\frac{1}{\log(1-x)}}$

(b) Find a and b , if $\lim_{x \rightarrow 0} \frac{a \sinh x + b \sin x}{2x^3} = \frac{8}{6}$.

- (B) Show that : [04]

$$x \operatorname{cosec} x = 1 + \frac{x^2}{6} + \frac{7}{360} x^4 + \dots$$

(C) Prove that n^{th} derivative of $y = \tan^{-1}x$ is

$$(-1)^{n-1} (n-1)! \sin n \left(\frac{\pi}{2} - y \right) \sin^n \left(\frac{\pi}{2} - y \right). \quad [04]$$

Q.5) Solve any two :

(a) If $u = \log(\sqrt{x^2 + y^2 + z^2})$,

then prove that

$$(x^2 + y^2 + z^2) \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) = 1. \quad [06]$$

(b) If $u = \frac{x^2 y^2 z^2}{x^2 + y^2 + z^2} + \cos \left(\frac{xy + yz}{x^2 + y^2 + z^2} \right)$,

then find the value of

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}. \quad [07]$$

(c) If $u = x^2 - y^2$, $v = 2xy$ and $z = f(u, v)$, then show that

$$x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = 2\sqrt{u^2 + v^2} \frac{\partial z}{\partial u}. \quad [06]$$

OR

Q.6) Solve any two :

(a) If $ux + vy = 0$ and $\frac{u}{x} + \frac{v}{y} = 1$,

then show that

$$\left(\frac{\partial u}{\partial x} \right)_y - \left(\frac{\partial v}{\partial y} \right)_x = \frac{x^2 + y^2}{y^2 - x^2}. \quad [06]$$

(b) If $u = \operatorname{cosec}^{-1} \left(\sqrt{\frac{x^{1/2} + y^{1/2}}{x^{1/3} + y^{1/3}}} \right)$,

then show that

$$x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = \frac{\tan u}{12} \left(\frac{13}{12} + \frac{\tan^2 u}{12} \right). \quad [07]$$

(c) If $x = r \cos \theta$, $y = r \sin \theta$, where r and θ are functions of t , then prove that :

$$x \frac{dy}{dt} - y \frac{dx}{dt} = r^2 \frac{d\theta}{dt}. \quad [06]$$

Q.7) (A) If $u = x(1 - y)$ and $v = xy$,

find $\frac{\partial(x, y)}{\partial(u, v)}$. [04]

(B) Examine for functional dependence for $u = y + z$,

$$v = x + 2z^2, w = x - 4yz - 2y^2. \quad [04]$$

(C) Discuss the maxima and minima of

$$f(x, y) = xy + a^3 \left(\frac{1}{x} + \frac{1}{y} \right). \quad [05]$$

OR

Q.8) (A) If $x = u^2 - v^2$, $y = uv$, find $\frac{\partial u}{\partial x}$. [04]

(B) Find the percentage error in computing the parallel resistance r of two resistances r_1 and r_2 from the formula $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2}$,

where r_1 and r_2 are both in error by 2% each. [04]

(C) Find the points on the surface $z^2 = xy + 1$ nearest to the origin, by using Lagrange's Method. [05]